Distributed Learning in Secondary Spectrum Sharing Graphical Game

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A presentation by

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Outline

1. Introduction
2. Game Theoretic Model Formulation
3. Learning and Punishment
4. Simulation Results
5. Conclusion and Future Works
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3. Learning and Deviation Punishment
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5. Conclusion
Introduction

Spectrum Sharing Literature
- El-Farol Game Family
  - Minority Game/Simplex Game
  - Local Minority Game
- Learning Concern: Can local inductive approaches solve the spectrum sharing game (global) optimally?
  - Deviation Punishment
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Spectrum Sharing Model

\[ \{ K = \arg\max_{k_i} \sum_{i} U_i(x_{ki}, I_{ki}, y_{ki}) \}, \]
\[ \forall i \in \{1, ..., n\}, \forall k_i \in \{1, ..., B\} \] (1)

- Secondary Users Connected to Primary Band 1
- Secondary Users Connected to Primary Band 2
- Secondary Users Connected to Primary Band 3

Primary Band 1, \( y_1 \)
Primary Band 2, \( y_2 \)
Primary Band 3, \( y_3 \)
The graphical spectrum selection game (GSSG) Elements

- Secondary users as the players, set of pure strategies for user (vertex) $i$ is the set of $S_i = \{1, ..., B\}$.

- Then the utility user $i$ receives, $v_i$, is given by:

$$v_i(k_i) = \sum_{j \in N(i)} w_{ij} M_i(k_i, k_j)$$

(2)

where $w_{ij}$ is the weight of interaction between player $i$ and $j$

- $M_i$ is realized by the following ant-coordination payoff matrix

$$
\begin{pmatrix}
0, 0 & y_1, y_2 & \cdots & y_1, y_B \\
y_2, y_1 & 0, 0 & \cdots & y_2, y_B \\
\vdots & \vdots & \ddots & \vdots \\
y_B, y_1 & y_B, y_2 & \cdots & 0, 0
\end{pmatrix}
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Nash - Evolutionary Stable Strategy (ESS)

- Potential Function Exists for the proposed spectrum sharing played on the **Weighted** graph $G$

Figure: Spectrum selection for $B = 3$ denoted by three different colors.

- Nash Equilibrium of the Anti-coordination Game $\mathcal{E} = [y_1, y_2, \ldots, y_B]$
Nash - Evolutionary Stable Strategy (ESS)

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![Figure](image)

**Figure**: Spectrum selection for $B = 3$ denoted by three different colors.

- Nash Equilibrium of the Anti-coordination Game
  
  $\mathcal{E} = [y_1, y_2, \ldots, y_B]$
Evolutionary Stable Strategy played on an appropriate graph under appropriate learning results in $X = \left[ \frac{1}{y_1}, \frac{1}{y_1}, \ldots, \frac{1}{y_B} \right]$

Utility function definition

$$U_i(x_{k_i}, I_i, y_{k_i}) = x_{k_i}y_{k_i}/(1 + I_i)$$  \hspace{1cm} (3)$$

Price of Anarchy Improvement

$$PoA(G) := \frac{\max_{s \in S} U(s)}{\min_{s \in E} U(s)}$$
Game Theoretic Model

- Evolutionary Stable Strategy played on an appropriate graph under appropriate learning results in $\mathcal{X} = [\frac{1}{y_1}, \frac{1}{y_1}, \ldots, \frac{1}{y_B}]$

- Utility function definition

$$U_i(\mathcal{X}_k, I_i^k, y_k) = \mathcal{X}_k y_k / (1 + I_i^k)$$  \hspace{1cm} (3)

- Price of Anarchy Improvement

$$PoA(G) := \frac{\max_{s \in S} U(s)}{\min_{s \in \mathcal{E}} U(s)}$$
Local Exponential Learning Algorithm

- Every node $i$ broadcast the chosen spectral band to its neighbors.
- Evaluate utility $v_i(s, t)$ using (2) $\forall s \in \{1, ..., B\}$.
- Update mixed strategy profile according to (4) & (5).
- Select spectral band using mixed strategy profile.
- Stop if: $\max_s |v_i(s, t) - v_i(s, t-1)| \leq \beta, \beta \geq 0$ $\forall i \in \{1, ..., n\}$. Otherwise go back to step 1.

$$p_{is}(t) = \frac{e^{\Gamma U_{is}(t)}}{\sum_{s' = 1}^{B} e^{\Gamma U_{is'}(t)}}$$, $\forall s \in \{1, ..., B\}$  \hspace{1cm} (4)

$$U_{is}(t + 1) = U_{is}(t) + v_i(s, t)$$  \hspace{1cm} (5)
The idea of punishing nodes can be by setting the weights $w_{ij}, j \in \mathcal{N}_i$ in the graphical game.

- $w_{ij} = 0$, it means that the secondary node $i$ ignores the outcome of interaction with user $j$.
- If $w_{ij} = -\infty, \forall j \in \mathcal{N}_i$ then all the neighbors punish node $i$ by jamming it, i.e; they transmit in the same band as $i$ w.p. 1.

This mimics the ‘Homo Reciprocans’ behavior.
Deviation Punishment

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- This mimics the ‘Homo Reciprocans’ behavior.
Congestion As ESS
Price of Anarchy

Simulation Results

- $B=4$, $Y=[1/4, 1/4, 1/4, 1/4]$
- $B=3$, $Y=[1/3, 1/3, 1/3]$
- $B=3$, $Y=[1/6, 1/3, 1/2]$
Dynamic Price of Anarchy

- B=5 with Dynamic Y value
- B=5 with Homogeneous Constant Y value
Network Volatility

Switching over Primary Bands

- B=5 with R=1
- B=5 with R=1.5
- B=5 with R=2
- B=4 with R=1
- B=4 with R=1.5
- B=4 with R=2
Deviation Punishment

![Graph showing Cheating User Payoff over Iteration](image)

- Cheating phase
- Punishment Phase

- Iteration
- Cheating User Payoff

- Payoff values: 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0
- Iteration range: 0 to 30
Conclusion & Future Works

- Ising model Improvement
- So many Unanswered Questions.
- Learning Improvement/Convergence Rate
- Punishment Ideas using Graphical Game Theory
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Thanks For your attention! Questions?